NOTE ON COMPLEMENTARY FRESNELLIAN FRINGES

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Measurement of Small Angles without Auxiliary Mirror.—This method makes use of the original apparatus heretofore described, but the two mirrors M and M' or N and N', figure 1, are rotated together as a rigid system around a vertical axis, at A for instance. In view of the absence of auxiliary reflection, the method will be but half as sensitive as the preceding one, so that the equation

$$R\Delta\alpha = \Delta N \cos i$$

is sufficient to approximately express the results. But on the other hand, if spectrum fringes are to be observed, there is greater abundance of light since the half silver film is penetrated but once by each component, *ac* or *bd*. When the achromatic fringes are used however the light is always superabundant and must be reduced in intensity. To try this method the mirrors N and N' were mounted on a good divided circle so as to rotate together on a rigid arm over a small angle α. The achromatic fringes displaced in this way were restored by advancing the mirror M' over the distance ΔN along the normal micrometer screw at n'. The following is an example of the results of corresponding values of ΔN and $\Delta\alpha$, when the distance apart of the rays *ac* and *bd* was $2R = 7$ cm.

$\alpha =$	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1.0°
$\Delta N \times 10^3 =$	0.0	8.0	16.6	35.6	34.9	42.7	50.7	61.7	70.5	78.5	90.6 cm.

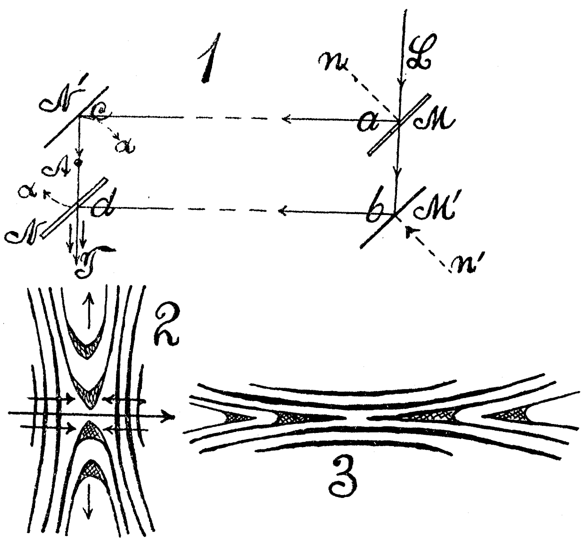
These results as a whole are much smoother than the earlier ones, for incidental reasons. From a graphic construction the mean rate $\Delta N/\Delta\alpha = 0.088$ cm/degree or 5.0 cm/radian may be obtained. Hence since $2R = 7$ cm., $(\Delta N/\Delta\alpha)2/R = 0.013$ in terms of degrees or 0.72 in terms of radians. This result is roughly half that in the preceding paper, if incidental errors be disregarded. From the above equation since $i = 45°$ nearly,

$$\frac{\Delta N}{\Delta\alpha} = \frac{2R}{2\cos i} = \frac{7.0}{1.41} = 5.0$$

agrees closely with the experimental result.

Complementary Fringes—Attention was now given to the hyperbolic fringes of a fine slit and white light, observed in the telescope at T when the ocular is drawn outward or to the rear of its position for the principal

focus and the spectroscope is removed. They appear and widen with the washed image of the slit. They are quite strong and sharp throughout and gorgeously colored, the fields and shades figure 2 being nearly complementary in color. The spectrum fringes must be centered if the others are to occur. The former (fig. 3) are usually long ellipses or hyperbolic with the major axis horizontal, while the corresponding new fringes (fig. 2) are hyperbolic with their major axis vertical. They are extremely sensitive to rotation of the micrometer mirror about a horizontal axis, rising or falling, but they soon vanish. When the micrometer mirror M' is rotated around a vertical axis, an operation which separates the white slit images in the telescope if originally coincident, the new fringes move bodily by displacement, from left to right, or the



reverse, depending on the sign of the rotation, while they continually change their color scheme. When the design is thus displaced as a whole the individual fringes move as shown in figure 2. As a group the fringes closely resemble the lemniscates of a binaxial crystal in polarized light. The variation of the color scheme is probably the same since with homogeneous light (sodium) the design is in yellow and black. The pattern is not quite dichroic but appears so, red-green, blue-yellow combinations with intermediate violet-yellowish succeeding each other. In polarized light the figures are sharpened as a whole, but there is no discrimination. The pattern gradually vanishes with a wide slit, whereupon the achromatic fringes may be seen when the ocular is restored to the principal focal plane.

If the white slit images in the telescope pass through each other (in consequence of the rotation of M' about a vertical axis, as specified) the direction of fringes *twice* changes sign in rapid succession and this probably occurs when the white slit images are coincident. Barring this inversion, the march is regular and proportional to the rotation.

With the displacement (ΔN) if the mirror M' on the micrometer screw normal to its face, the fringes pass through a continuous succession of color schemes, but soon vanish; for they coincide in adjustment with the centered spectrum fringes. Similarly if a pair of mirrors (MM' or NN' (fig. 1)) rotates about a vertical axis as a rigid system, the same continuous change of color scheme and evanescence is apparent.

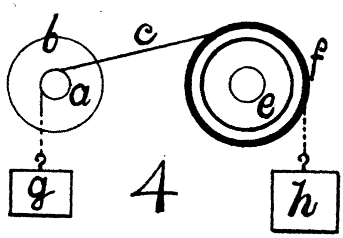
These interferences differ, naturally, from the spectrum interferences; they also differ from the achromatic interferences, which are much finer fringes, partaking of the regular fringe pattern seen with biprisms. They are a separate phenomenon, quite sharp and definite, occurring under like conditions of adjustment, but under different conditions of observation (ocular out of focus and fine slit). In the principal focus the two sharp extremely bright slit images are alone present. They are absolutely identical in structure, however, and their spectra when superposed would interfere symmetrically throughout their extent. Under these circumstances the rays intersecting in the white slit images also interfere before and behind the principal focal plane of the telescopic images specified, and this interference is not destroyed when the slit images are separated (rotation of opaque mirror M' about vertical axis), or when the slit images are passed through each other. What is not at once seen however is the reason of the occurrence of large sharp definite hyperbolic forms instead of the usual Young or Fresnel fringes of two slits or slit images.

On the Michelson interferometer these fringes (like the achromatic fringes) are extremely faint and can hardly be detected except by putting them in slow motion. The spectrum fringes are equally strong in all cases. It appears therefore that the two half silvers are favorable to evolving both the hyperblic and the achromatic set of fringes. The Michelson design is thus not useful here, nor for the measurement of small angles of rotation by the methods described, as the mirrors would have to be rotated in opposite directions.

Further work with the complementary fringes on different interferometers of the Jamin type showed that to produce the hyperbolics, the fine slit images must coincide horizontally and vertically. They do not in this case probably coincide in the fore and aft direction; for the plates, etc., were not optically flat. When the slit images are separated at the

same horizontal level into two fine parallel lines, the complementary fringes in fact become Fresnellian fringes, finer as the slit images are more separated and as the ocular is more rearward or forward. This is precisely what should occur. We may conclude therefore that the complementary fringes are Fesnellian interferences of two slit images and that the central hyperbolic forms are due to outstanding front and rear positions of the two slit images, which seem to coincide in the field of the telescope. Differentiated from these, the achromatic fringes are referable to the colors of thin plates. I have in fact since succeeded in obtaining the complementary fringes in the shape of broad straight gorgeously colored vertical bars, without suggestion of hyperbolic contour.

An attempt was made to get quantitative estimates of the passage of fringes, on rotating the paired mirrors NN' over an angle α. To control the small angles the device figure 4 was improvised and did good service. Here e is the tangent screw of a 6″ divided circle. It is surrounded by a snug anulus of cork and holds the brass ring f, on whose surface a



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coarse screw thread has been cut. Near this and with its axis in parallel is a $\frac{1}{4}$ inch screw a and socket (not shown), controlled by the disc b. A strong linen thread c is looped once around f in the grooves of the screw and once or twice around the grooves of a, the string being normal to the cylinders and kept taut by two small weights, g, about a half ounce and h, about an ounce. The head b may be turned either way and the angle read off in minutes on the head of the tangent screw e.

The theoretical value apart from glass paths and other corrections should be, per fringe vanishing

$$2R\Delta\alpha = \lambda$$

where R is the radius of rotation corresponding to the angle $\Delta\alpha$ and λ the mean wave length of light. In the given adjustment $2R = 10$ cm. was the normal distance apart of the two interfering beams. Hence

$$\Delta\alpha = \frac{\lambda}{2R} = \frac{60 \times 10^{-6}}{10} = 6 \times 10^{-6} \text{ radians or } 1.2'',$$

per vanishing fringe. As the change of glass path of one beam would have to be deducted from $2R$, a somewhat larger value would be anticipated. Testing the complementary fringes (white light) the passage of about 25 fringes completed the phenomenon after which it paled to whiteness. These 25 fringes passed within $\Delta\alpha = 0.75'$, or per fringe about $0.03'$ or $1.8''$ of arc. Of course this is merely an estimate from the small angles of turn involved.

The complementary fringes with sodium light are available indefinitely. I counted about 100 fringes for an angle of $2.7'$, i.e., $1.6''$ per fringe.

Finally using the spectrum fringes of the spectroscope, about 120 fringes were counted within $3'$, i.e., $1.5''$ per fringe. All of these values are larger than the computed value $\lambda/2R$ without correction, but in view of the large number of fringes within exceedingly small angle $\Delta\alpha$, sharp agreement is not to be expected.

[This note is from a Report to the Carnegie Institution of Washington, now in preparation.]

THE DISPLACEMENT INTERFEROMETRY OF LONG DISTANCES

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1. *Small Angles.*—In my preceding notes I suggested two methods for the measurement of small angles. The first used an auxiliary mirror and apart from corrections the angle $\Delta\alpha$ over which the auxiliary mirror turns is

$$\Delta\alpha = \Delta N \cos i/2R \tag{1}$$

where ΔN is the displacement of one of the plane mirrors parallel to itself necessary to restore the achromatic fringes to their former position in the field of the telescope, i the angle of incidence (conveniently 45°), $2R$ the normal distance apart of the (parallel) interfering pencils in the fore and aft direction of the incident beam. In the second method the auxiliary mirror is dispensed with and the rotation of a rigid system of paired mirrors is used. The sensitiveness is half the preceding.

2. *Distances.*—Suppose now that the paired mirrors near the telescope confront but a part of the area of the objective and that the telescope can therefore look over the mirrors directly into the region beyond. (A series of small mirrors or reflecting prisms may be employed